ARITHMETIC SIMPLIFIED
FOR GENERAL USE

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ARITHMETIC SIMPLIFIED

FOR GENERAL USE

AND ADAPTED TO AID STUDENTS ENGAGED IN ANY
DEPARTMENTS OF SCIENCE OR ART,

ALSO

TO SERVE AS A SUPPLEMENT TO THE AUTHOR'S

ELEMENTS OF PHYSICS,

AND OTHER WORKS ON POPULAR SCIENCE.

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ARITHMETIC SIMPLIFIED

FOR GENERAL USE.

To the Reader.

The first thirty-two pages of this Tract treat of the Arithmetic. The last few, from page 33, give direction deemed important, for the study of Science generally, and should be read first. The Tract may conveniently, where desired, be bound into the volume of Physics.

Formulæ in Science.

The following pages exhibit the abridged view of arithmetic promised in the last paragraph of page 716 of the Elements of Physics, published in 1865.

What is here offered is not a complete or exhaustive Treatise on Arithmetic, but an endeavour to remove or lessen certain difficulties which common minds encounter in the attempt to go well into the subject; and the author hopes that this simple introduction will so far lessen these difficulties that the number of successful students will be thereby much increased.

Formulæ are short arithmetical expressions showing general relations among the phenomena of nature; a knowledge of these greatly facilitates the computations required in particular cases, and they are therefore important to be understood by all who make applications of science to practical purposes.

As some of the signs or symbols used in these are employed also in algebra, many persons who have not learned algebra avoid the study of formulæ because of the supposed difficulty; but those who know the common arithmetical operations of addition, subtraction, multiplication, and division, with the signification of the few marks explained, at page 707, and in common books, are prepared to understand all that follows here.

§ 1. A chief cause why so many people have imperfect conceptions and command of the powers of common arithmetic is, that in ordinary schools the methcds of teaching arithmetic

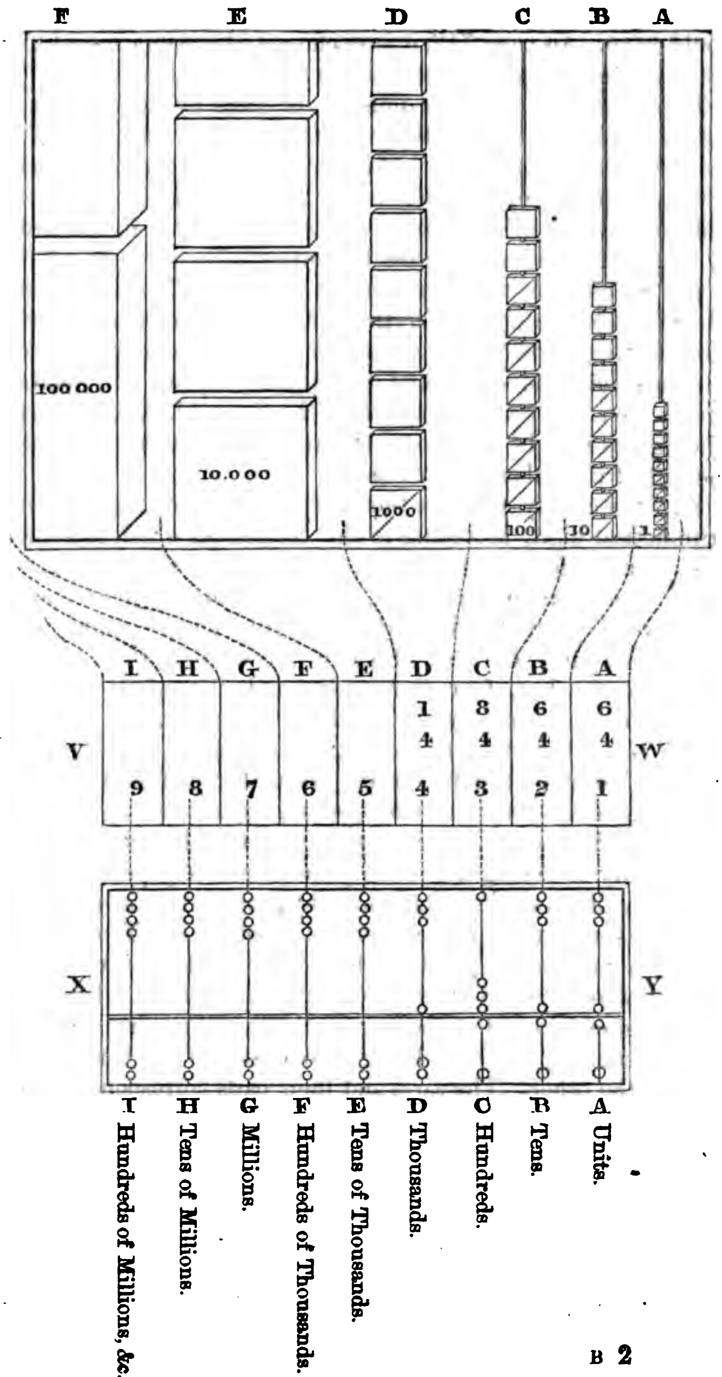
do not begin at the true beginning. Pupils learn to practise mechanically rules for multiplying, dividing, working the rule of three, and so forth; but the reasons of the rules are so imperfectly communicated, that many misconceive them, and, in consequence, soon forget and misapply them.

§ 2. A prime fact not sufficiently impressed on the learner's mind, is, that all ordinary computations are the comparing of numbers in general with some one convenient number chosen as a standard. That number is TEN, most convenient because it is the number of the human fingers, always present whenever men have to count, and as naturally adopted for numbers as the length of the human foot has been for lengths.

§ 3. The crossed-square exhibited and explained above at page 703, presents to the eye, and thereby very clearly to the mind, many important relations among themselves of this number ten with its multiples or powers, and a learner profits much, who by frequent review and reflection makes it familiar to the thoughts. Some of the individuals who at an early age have become remarkable for their readiness in mental arithmetic, have owed their ability to the accident which led to their acquiring familiarity with some such standard arrangement of units.

§ 4. Another arrangement of kindred nature and utility is sketched in the adjoining page, which pictures clearly to the eye with appearance of solid bulk, what the crossed-plane shows in merely extended surface. It exhibits a succession of columns increasing in magnitude, consisting each of ten equal parts. In the first column, marked A, the parts are of very small size. Those of the second column, B, are ten times larger, one of its unit-parts being equal to ten units of the column A. Those of the third column, C, are in the same proportion larger than those of B, and so on in succession, a part in any column being equal to ten of those in the preceding column, and being only a tenth of one in the following column on the left. The number of primary units in a cube of each row is inscribed on the bottom one, viz., 1, 10, 100, 1000, 10,000, &c. The cubes of the seventh column, G, not shown here, would contain each a million of the primary units.

§ 5. The drawing represents the smaller columns entire; but of the larger, only the bottom cubes find room in the page, and the very largest do not appear there at all. The parts are represented as cubes nearly of the comparative sizes which the realities


F E D C B A
100 000
10.000
1000
100
10
1
I H G F E D C B A
1 8 6 6
4 4 4 4
9 8 7 6 5 4 3 2 1
V
W
X
Y
A Units.
B Tens.
C Hundreds.
D Thousands.
E Tens of Thousands.
F Hundreds of Thousands.
G Millions.
H Tens of Millions.
I Hundreds of Millions, &c.

would have. In page 712, it is explained why many solid bulks heaped closely together seem to occupy little space.

To express any particular number of units by this apparatus, the required number of cubes would be in some way marked in the different columns; for instance, the year of the present era, 1866, is signified by one cube in the site D, eight in the site C, and six in each of the sites B and A, as indicated in the figure by diagonal lines drawn across the cubes.

§ 6. If beneath the arrangement of cubes shown, the lines which divide the spaces for the different columns, are carried down to form corresponding spaces for the common arithmetical figures below, the cubes and the figures explain one another. This is seen in the engraving where the figures 1, 8, 6, 6, appear. The figures may be considered as miniature pictures or signs of the sets of the cubes marked above.

§ 7. It is clearly seen how the difference of value is given to an arithmetical digit, according to the column in which it stands. The figure 4, for instance, when alone, means simply four, but when several are placed thus, 4444 in a line, as belonging to the several columns, the extreme figure on the right hand means four units, the next to it four tens or forty, the next four hundreds, the last, four thousands; and the whole together signify four thousand four hundred and forty-four.

§ 8. Near the bottom of the diagram in page 3, is represented still another mode of representing decimal sets of numbers. It is that used all over the Chinese empire, and the apparatus is called the *Swan-pan*. The present writer had in early life the opportunity of seeing how readily and accurately persons of all classes in China made calculations by means of this swan-pan. It consists, as here shown between the letters X and Y, of a frame having a number of parallel wires crossing it, on which wires balls of wood or ivory slide like beads on a cord. A bar, X Y, runs from end to end of the frame, dividing the wires and their balls unequally, two balls being on one side of the bar and four balls on the other. The wires and their balls correspond, as here shown, to the columns A, B, C, D, E, F, of the cubes above, and to the arithmetical figures or digits in the middle part of the diagram, between the letters V and W. The four balls are used to signify each a single unit of the different orders, the two balls signify each five such units. The balls are significant only when moved into contact with the bar. Here the balls represented as in contact with the bar indicate the present year

of our era, 1866; the one ball near the bar, of the rank D, marks 1000; the three balls of single units, with the one ball of five on the wire C, mark eight hundreds; and the one ball of unit value with one of five-fold value on the wires B and A mark six for each; the whole expressing 1866.

§ 9. To facilitate reference to the succession of columns or ranks, which determine the different values of the figures placed in them, the adjoining table is given, showing the relation of the figures and the letters to the names written at length.

10	9	8	7	6	5	4	3	2	1
K	I	H	G	F	E	D	C	B	A
&c.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.

§ 10. The explanations here offered show, that any number of units which the mind can conceive may be expressed with absolute precision by any of the modes described, and much better than was done formerly by the Greeks and Romans, who used the letters of their alphabets; but the ten Indian figures or symbols 1, 2, 3, 4, &c., far surpass the others for the purposes to be served. The aid given by them to profound computations is singularly great, and the compendious form of the written expression is equally remarkable. As exemplifying the last quality, one may see on a single page of a small book, clearly set forth, such a mass of statistical information as the following titles indicate, many of the facts being results of wide observation and great labour of calculation:—

The amount of population of a great city.
The amount of population of a kingdom.
The amount of population of the world.

The number of persons in a community alive at different ages.

The number of persons employed in the special kinds of employment exercised in a society.

The comparative fatality of different diseases.

The comparative duration of life in different countries.

The rate of increase of populations. For instance, the fact is now ascertained, that the mixed race called Anglo-Saxon in North America has doubled in number every twenty-five years for at least four periods, and, if the laws of nature continue unchanged, the increase at the same rate will go on for at least four periods more, before wider diffusion becomes a necessity.

At present there are more than thirty millions of the race	30	millions
In the first period there will be . .	60	„
In the second	120	„
In the third	240	„
In the fourth (within a century hence) .	480	„

which is about a third of the present population of this globe. Then North America shows but one of the European colonies that are now similarly progressing, chiefly because of men's advanced knowledge of the universe, and the many new arts sprung from such knowledge during the last two centuries. It is calculated that the portion of the earth's surface on which the rain falls that fills the channels of the rivers Mississippi and St. Lawrence, could, if completely cultivated and managed according to modern skill, supply the necessaries and even luxuries of life to the whole present population of the world. Such are among the astounding facts which men of the present day have to contemplate; and they may see that the other arts which have worked the change, as that of navigation founded on astronomy, are mainly due to improvements in the means of measuring and computing.

§ 11. It is known that Charles Babbage, who is still living in health amongst us, after leaving the university, invented a machine, now known as Babbage's calculating machine, which, when moved by the turning of a crank or otherwise, is made to calculate and even to set the types and to print, without possibility of error, mathematical and other tables of the highest importance. In former times such tables had to be produced under the superintendence of governments, with enormous labour and cost, and without insuring accuracy.

Much of Mr. Babbage's merit lay in the discovery that the operations of adding together any two of the ten Indian ciphers, and carrying a unit to a higher column of figures, whenever the sum of the added ciphers reached or exceeded ten, was the essential requirement in the most complex calculations—and this operation his machine can unfailingly execute.

It would exceed the assigned limits of the present sketch to enter into the details of Mr. Babbage's invention. Many persons have now studied it, and two ingenious Swedish Engineers, Messrs. Scheutz, father and son, have been able to simplify the mechanical arrangement which performs the work. The object of the present sketch is to explain, that through the possession of the ten ciphers or arithmetical symbols now common, with the aid of pen and paper and simple directions for using the symbols, a computer has to attend closely to only two or three ciphers at a time, chiefly adding and subtracting and carrying tens; so that his mind is converted into a useful calculating machine, doing easy and correct work almost like that devised by Babbage.

§ 12. Besides the higher objects of making various scientific computations, the utility to mankind generally of having a fair amount of practical familiarity with counting and measuring, can hardly be overrated. With such knowledge, a carpenter can form more accurately his square boxes; a mason can erect better his upright walls; a mariner can steer his course more safely across the trackless waves. For want of such knowledge, persons, generally of the labouring classes, dependent for the support of their families on wages received at short intervals, are suffering multiplied difficulties and distress. These, and many others, would escape much of the unhappiness now common in the world, if they were early trained to the habit of balancing accounts of income and expenditure, as the basis of prudence and economy. It has been remarked that persons of the labouring classes, who once begin to keep accounts of expenditure, and to lay by a portion, however small, of their earnings, rarely come to want.

§ 13. There are but four fundamental operations in arithmetic; of these, separately, or in combination, the higher operations of computing and measuring consist, namely:—

Addition,
Subtraction,
Multiplication,
Division.

§ 14. Addition

Is the joining together, or fusing into one sum, two or more separate numbers.

A ferry-boat may carry across a river in succession, the numbers of passengers here noted. A reckoner, having written the figures in a vertical row, might begin counting from the bottom, and say—6 and 6 make 12, and 5 make 17, and 9 make 26, and 4 make 30, and lastly, 8 make 38, or three tens and eight ones or units.

B	A
	8
	4
	9
	5
	6
	6
3	8

After moderate practice in such work, the process is carried on almost as readily as when a person in reading connects the letters of printed lines into words.

§ 15. When the numbers dealt with are large, the ciphers are placed in distinct columns, as shown below, to be added and carried separately. For easy reference, lines are placed between the columns, with a letter at the top of every column. The reckoner begins at the bottom on the right hand, and finds in the column A, 18 units, or one ten and eight units over. The excess of units above ten is written at the bottom of the column, and the one ten is carried forward to the next column, B. This one, when added to the 16 tens already there, makes 17, of which the excess over ten is written at the bottom, and the ten of tens is carried a step higher to the column C, which is of hundreds; and so the computer counts onward to whatever extent the numbering, in this way, is carried. The sum total in the example here given is ten thousand three hundred and seventy-eight.

E	D	C	B	A
		6	2	2
	1	9	3	5
	2	6	0	3
	3	0	4	8
	2	1	7	0
1	0	3	7	8

It is so useful to the student to be able to distinguish clearly the different columns, that this mode of illustration is continued in subsequent lessons.

§ 16. It is to be observed here that the cipher or zero (0) has no numerical meaning in itself, but serves to keep the other figures in their proper columns towards the left side.

§ 17. For general arithmetical counting, the carrying or standard number in summing up is *ten*. It would be an important simplification of the whole business of calculation, whether in science or for purposes of commerce, as in relation to money, weights, measures, &c., if the decimal system of subdividing

large amounts and grouping smaller were adopted universally. A change towards this result is steadily progressing. Hitherto different countries have from accident adopted different systems. In England, for instance, the subdivision of money has long been into pounds, shillings, and pence, of which 12 pence form a shilling, and 20 shillings a pound. In adding amounts of money, therefore, the carrying number from pence to shillings is 12, and from shillings to pounds is 20, as exemplified in the adjoining example of four sums united into one.

£	*s.*	*d.*	
4	18	6	
2	3	4	
5	9	3	
1	12	10	
£14	3	11	The sum.

§ 18. Subtraction.

This is an operation the reverse of addition, telling how much is left when one number is taken from another.

In working the two horizontal lines of figures are placed one above the other, as in addition, the larger uppermost. Then, instead of joining the two figures of the same column into one *sum*, as in addition, the *difference* of the two is noted and written at the bottom of the column. Here, in the column A of units, 4 is taken from 8, leaving the remainder or difference, 4, which is then written below. In column B of tens, 9 below has to be taken from 6 above, but as this cannot be done, a unit from the higher column, C, is borrowed, which makes 16 in the column B, from which the 9 being taken, 7 remain to be noted below. The reckoner then passing to the column C, has to deduct one (1) from the 2 above it, but this 2 having been now reduced to 1 by the borrowing for the B column, when another 1 is taken away, nothing is left. A zero (0) is therefore written below. The reckoner then proceeds to the column D, and taking 7 from 9, has 2 left to be written at the bottom of the D column. In the column E, there being no figure below to be subtracted, the figure 3 is carried down.

E	D	C	B	A
3	9	2	6	8
	7	1	9	4
3	2	0	7	4

§ 19. Multiplication.

B	A
	8
	6
4	8

This is the operation, which, taking any number to be called the *multiplicand* (Latin for "what is to be multiplied"), and any other number to be called the *multiplier*, finds what sum or *product* arises when the multiplicand is added to itself as many times as there are units in the multiplier. Thus, if the multiplicand be 8 and the multiplier 6, the product is 48, for 6 times 8 makes 48. This operation may be considered as merely an abridged form of addition, and may be thus exhibited.

B	A	
	8	Units.
	8	added to 8 make
	8	„ 16 „
	8	„ 24 „
	8	„ 32 „
	8	„ 40 „
4	8	The sum.

B	A
1	6
2	4
3	2
4	0
4	8

§ 20. The process of multiplication is singularly abridged and facilitated when the student has committed to memory the so-called *multiplication-table* here exhibited. This may be regarded as a portion of the *crossed-square* described in page 703, having its divisions numbered at the boundary lines, commencing from one corner as A. At the meeting of the rows of small squares, proceeding inwards at right angles from the border lines, there is placed the product of the two border numbers. Thus, at the meeting of the rows, from the numbers 6 at the top and 6 on the side, the small square has the number 36 marked on it, which is the product of 6 times 6. Again, where the border numbers are 8 above, and 9 on the side, the lines meeting within show the number 72, which is the product of 8 and 9; and so for all other combinations.

In this table proof is readily found of the following important facts.

§ 21. If two numbers (to be called *factors*) are multiplied together, it makes no difference which of them is taken first, that is to say, which is deemed multiplicand and which multi-

plier. For instance, the student's eye is led towards the same product, 63, whether he begins at the boundary from 7 or from 9, making 7 times 9 or 9 times 7.

§ 22. The space or area enclosed by any two lengths of the boundary lines, and corresponding lengths of parallel lines within, which space is always a rectangle, or that figure which has all its corners right angles, is exactly proportioned in

THE MULTIPLICATION TABLE.

A

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

extent to the product of the two numbers at the extremities of the measuring lines. For instance, within the space bounded by the lines which meet within from the numbers 7 and 9, on the outside, there are just 63 small squares or equal portions of surface. Within the whole square, bounded by the lines A 12 on the top and A 12 on the side, there are just 144 of such small surfaces, and 12 times 12 make 144 in numbers.

§ 23. The exact relation thus discovered between the lengths of straight lines and the amount of surface bounded by them is a matter of high importance, and it further leads to the

determination of the relation between length of lines and the volumes or solid bulks bounded by them—for if 144 small cubes like dice are placed to cover the 144 small squares of a multiplication table, twelve times that number, viz., 1728, when placed as similar layers one above another, produce an exactly cubical mass, having an edge of 12 of the lineal measures.

§ 24. The table shows also that a *diagonal* or straight line drawn between opposite corners of any rectangle cuts it exactly into halves, for if one part be lifted and laid down upon the other, all the sides and angles of the two perfectly coincide.

§ 25. To perform multiplication, the practice is to place the figures of the multiplicand, as here shown, in a line with the unit of the multiplier directly under the unit of the multiplicand, and then to multiply by it in succession the figures of the multiplicand, carrying the number of tens produced by every pair of factors to the more advanced column on the left hand, while any excess above tens is noted in the column below.

F	E	D	C	B	A
	8	5	0	6	2 4
3	4	0	2	4	8

In the example here given, the computer says, 4 times 2 make 8, which is written below, then 4 times 6 are 24, of which the 4 B units are written down, and the two are carried to appear in the C column, where there is nothing else, as the 0 fills the space in the multiplicand rank. Then 4 times 5 make 20, for the D rank, 0 is written and the two tens carried. Then 4 times 8 make 32, which, with the two carried from the D rank make 34 for E, of which the four units are noted below, and the three tens appear in the column F. The whole product is three hundred and forty thousand two hundred and forty-eight.

§ 26. To operate when both multiplicand and multiplier consist of several digits, the process becomes as here shown.

G	F	E	D	C	B	A	The letters distinguishing the columns.
			3	7	4	1	The multiplicand.
				4	2	3	The multiplier.
		1	1	2	2	3	The partial product by 3 units.
		7	4	8	2		,, ,, by 2 tens.
1	4	9	6	4			,, ,, by 4 hundreds.
1	5	8	2	4	4	3	General product.

It is seen here that the operation is divided into three distinct parts by using the three figures of the multiplier separately, beginning with the unit 3 on the right hand. This gives the product, as read from right to left, 32211; then the second or ten-digit of the multiplier 2 gives products of higher ranks 2847; and lastly, the figure 4 (hundreds) gives 46941. These three products added together make the general result 1582443, which, in words, is one million five hundred and eighty-two thousand four hundred and forty-three.

Here, as elsewhere, the remark applies that, although the numbers are great and the process complicated, the computer has to attend closely to only two or three of the ten arithmetical symbols at one time, and to follow a simple rule or direction which guards against his falling into error.

§ 27. Division.

This operation consists in ascertaining how many times or parts of a time, in a quantity to be called the *dividend* (the Latin for "that which is to be divided"), a certain other number to be called the *divisor* ("that which divides") is contained; and the number which declares the *how often*, is called the *quotient* (from the Latin *quoties*, "how often"). The common form of statement is this:—

Divisor.		Dividend.		Quotient.
8	)	48	(	6

The number 8 is contained in 48 six times—as, six times 8 make 48.

If multiplication is but abridged addition, division is abridged subtraction, and the column of figures exhibited in page 10, read in the contrary direction, serves to illustrate the relation. The number 8 withdrawn from 48, by six successive subtractions, exhausts that number, leaving no remainder.

§ 28. Where both dividend and divisor are single figures or do not exceed 12, the common multiplication-table answers all questions in division. Where both consist of several figures, it becomes necessary, as in multiplication, to separate the dividend into parts, giving partial quotients, which, when finally added together, become the general quotient. This is well illustrated

by the simple case of dividing 963 by 3. The number 963 may be written thus:

	C	B	A		C	B	A
Hundreds	9	0	0	This contains 3 just 300 times	3		
Tens. . .		6	0	This „ 3 „ 20 „		2	
Units . .			3	This „ 3 „ once			1
Sum	9	6	3	Quotient	3	2	1

Or the process may be written thus:

	C	B	A
3)	9	6	3
	3	2	1

In this form the computer mentally separates the dividend into the parts of hundreds (C), tens (B), and units (A), each of which gives a partial quotient, which is written down underneath it, the figures being placed so as to show to what column or rank each belongs.

And a larger number, with divisor of a single digit, may be divided in the same way.

	F	E	D	C	B	A	
Divisor 4)	9	8	9	2	0	8	Dividend
	2	4	7	3	0	2	Quotient

The computer here says 4 in 9 twice and 1 over, which one F added to 8 E's makes 18 E's, in which 4 exists 4 times with 2 E's over; these two added to 9 D's make 29, in which 4 exists seven times with one D over; this one D added to 2 C's makes 12, in which 4 exists exactly 3 times, with no excess to carry. In the row B it does not exist at all, and 0 is written; and lastly, out of 8 A's it comes twice, as is marked in the quotient. Thus the fourth part of 989208 is 247302.

§ 29. The following example exhibits the common mode of performing division, where both dividend and divisor consist of

several figures. It shows the dividend visibly brought down in successive parts, each of which gives its own quotient to become part of the general quotient.

Divisor.

B	A
2	7

Dividend.

	E	D	C	B	A
)	6	7	3	6	5
†	5	4	.	.	.
	1	3	3	.	.
†	1	0	8	.	.
		2	5	6	.
†		2	4	3	.
			1	3	5
†			1	3	5
				0	0

Quotient.

	E	D	C	B	A
(		2	4	9	5

Partial products of the quotient figures and the divisor—reproducing, when added, the dividend.

†	5	4			
†	1	0	8		
†		2	4	3	
†			1	3	5
	6	7	3	6	5

Separate products subtracted.

The computer finds—

1. That the divisor 27, not being contained in the first single figure of the dividend, has to be taken from the first two, 67 E D, in which it is found twice. He accordingly writes 2 as the first figure of the quotient in the rank D.

2. He then subtracts twice 27 (54) from the part of the dividend first taken, leaving 13 D's as a remainder.

3. To this he brings down the next figure, 3 C, of the dividend, and adds it to the 13 left, making 133 C then to be divided.

4. In this he finds the dividend 4 times (making 108), which 4 he writes in the quotient 4 C, and subtracts 108 from the diminishing dividend, leaving a remainder of 25 C.

5. He then brings down, and adds to that remainder, the next part of the dividend, 6 B, making 256 B, which, containing the divisor nine times (243), he adds 9 B to the quotient.

6. He subtracts this product from the remaining dividend, and has a remainder 13.

7. To this he brings down the last figure of the dividend 5, making 135 units, in which the divisor is found five times without a remainder. He accordingly adds 5 to the quotient, and

completes the operation, finding that 27, the divisor, is found 2495 times in the dividend 67,365.

It is seen below the columns of the quotient, that the products of the divisor and of the single figures of the quotient, subtracted in succession from the dividend (and distinguished here by small crosses), when added together, as is done here separately, reproduce exactly the amount of the dividend.

§ 30. Fractions.

In the examples of division hitherto given, the dividends have all contained the divisors an exact number of times; as, when 12 pounds sterling are to be divided among 3 persons, and each person receives 4 pounds. But in the majority of cases which offer in practice, this coincidence does not occur. It might be required, for instance, to divide equally 13 pounds sterling among 3 persons. Evidently, when each had received 4 pounds, there would still be one pound left, of which, to complete the desired division, each person would have to get a third part. Now in Britain it happens that by law the pound is divided into 20 shillings, and the shilling into 12 pence, so that in the case supposed, the division is possible by giving to each person six shillings and eight pence—the well-known fee of a law-agent. But such coincidences are wanting in innumerable instances both of science and of practice. The subdivisions of wholes into smaller equal parts, whether really made or only conceived, are called broken numbers or *Fractions*, and the operations connected with them form a very important department of the art of computing. This is now to be considered, under the heads of *Vulgar* and *Decimal Fractions.*

§ 31. Vulgar or Common Fractions.

The term Vulgar does not imply, as might at first be supposed, something mean or of small moment, for the fact is the reverse, but is used, in the Latin sense of the word, for what is *common.* It serves also to distinguish common from decimal fractions—a modification introduced in modern time.

In the ordinary business of life, the necessity is constantly occurring of having to divide whole quantities or integers into the simplest subdivisions of halves, thirds, quarters, &c.; and no

words are more familiar than the names of such relations, but as science and arts advance, men have to speak of tenths, hundredths, thousandths; and in some cases even millionths are of consequence.

Fractions are commonly written by the mark of *division*, placing one number above a short line and another below it, thus $\frac{1}{2}$ is one half, $\frac{2}{4}$ two fourths or quarters, $\frac{6}{10}$ six tenths, and so on. The number below signifies into how many equal parts a whole thing is supposed to be divided, and the number above shows how many of these parts are taken to make the fraction. This explains why the number below is called the *denominator* of the fraction, and the number above the *numerator*. It shows us why a clear notion of the mutual relation of two numbers is given when they are connected in the form of a fraction.

As the mind can conceive a straight line to be divided into any number of equal parts, so can it conceive any magnitude or quantity whatever to be similarly divided.

With respect to fractions generally, the most ordinary apprehension can perceive that the following statements must be true.

§ 32. (*a*) If the numerator and denominator of a fraction are equal, as in the case of $\frac{2}{2}$ (two halves), $\frac{3}{3}$ (three thirds), $\frac{10}{10}$ (ten tenths), $\frac{100}{100}$ (a hundred hundredths), the value of the fraction is that of one whole number or integer; and all such fractions are fractionally equal to one another, that is, give the same quotient, namely one (1).

§ 33. (*b*) If both terms (numerator and denominator) of a fraction are either multiplied or divided by the same number, the value of the fraction is not changed. Thus $\frac{2}{3}$ so multiplied by 2 becomes $\frac{4}{6}$ (four sixths), equal to $\frac{2}{3}$—if multiplied by 4 it becomes $\frac{8}{12}$, if by 10 it becomes $\frac{20}{30}$, and so on, all fractionally equal. And, again, $\frac{12}{24}$ (twelve twenty-fourths) with both terms divided by 4 becomes $\frac{3}{6}$ (three sixths); so divided by 3 it becomes $\frac{4}{8}$ (four eighths), divided by 2 becomes $\frac{6}{12}$, and so on, all being equal.

§ 34. (*c*) This fact of reducing to a common denominator, gives the power of comparing exactly the value of any two fractions, however dissimilar. For instance, a student does not see at once the comparative values of $\frac{6}{7}$ and $\frac{8}{9}$, but if both terms of each be multiplied by the denominator of the other, the first becomes $\frac{54}{63}$, and the second $\frac{56}{63}$, and the difference between them is $\frac{2}{63}$; or they are to each other as 54 to 56.

The fractions $\frac{2}{3}$ and $\frac{3}{4}$ having more simple figures, if so treated. become $\frac{8}{12}$ and $\frac{9}{12}$, and therefore differ by $\frac{1}{12}$.

§ 35. (*d*) If the numerator only, of a fraction, be multiplied by any number, the value of the whole fraction is multiplied by that number; thus $\frac{3}{4}$, having its numerator multiplied by 8 becomes $\frac{24}{4}$, or six wholes. On the other hand, if the denominator only of a fraction be multiplied by any number, the value of the fraction is divided or diminished according to that number; thus $\frac{3}{4}$, having its denominator multiplied by 2 is reduced to $\frac{3}{8}$ (three eighths), or becomes just one half of $\frac{3}{4}$ (three quarters).

§ 36. (*e*) It is easier to operate with small numbers than with large, and a fraction may be reduced to lower terms without changing its value, by dividing both terms by any number which will divide both without leaving a remainder. Such number is called a common measure. Thus the numerator and denominator of $\frac{9}{27}$ so divided by 9 becomes $\frac{1}{3}$, and $\frac{75}{100}$ divided by 25 becomes $\frac{3}{4}$.

The greatest common measure of two numbers is found by dividing the larger number by the smaller, and if there be a remainder, by repeating the process of dividing the preceding divisor by the last remainder until no remainder is left. Thus, for $\frac{72}{108}$

```
72)108(1
    72
    --
    36)72(2
       72
       --
        0
```

It is now seen that the value of a fraction depends not at all upon the magnitude of its terms, but upon the numerical relation between them.

§ 37. A fraction of which the numerator is smaller than the denominator is called a *proper fraction*, as $\frac{3}{4}$. One of which the numerator is the larger term, as $\frac{12}{4}$ (which is equal to three wholes) (3), is called an *improper fraction*. To any whole number the fractional form is given without changing the value, by placing, as here, a short line of division under it, and the figure 1 below that as a denominator; as $\frac{3}{1}$ is three ones.

A whole number with a fraction added to it; thus $3\frac{1}{4}$ (three and a fourth) is called a mixed number, and may be transformed into a simple fraction by multiplying the integer part by the denominator and then adding, thus $\frac{13}{4}$ (thirteen fourths).

Fractions admit of the four fundamental operations of addition, subtraction, multiplication, and division, as wholes or integers do.

§ 38. Addition of Common Fractions.

It is evident that if several fractions have the same denominator, as $\frac{1}{4}$ $\frac{3}{4}$ $\frac{20}{4}$ (one fourth, three fourths, twenty fourths), to add them together is merely to add the numerators, retaining the denominator unchanged. In the example just given, the numerators are $1 + 3 + 20$, in all 24, which, with the unchanged denominator 4, make the fraction $\frac{24}{4}$ (twenty-four fourths) equal therefore to six wholes, $\frac{24}{4} = \frac{6}{1} = 6$.

To add together the values of unlike fractions, it is required, therefore, only to reduce all to the condition of having the same denominator, and then to add together the numerators.

§ 39. Subtraction of Common Fractions.

The rule is to reduce them to like denominators, as for addition, and then to subtract the one numerator from the other. In the last example given, of $\frac{2}{4}$ and $\frac{3}{10}$, the reduced fractions are $\frac{20}{40}$ and $\frac{12}{40}$, and the numerator of the one subtracted from that of the other (20 - 12) leaves $\frac{8}{40}$, which, reduced to the lowest terms by dividing each term by 8, is found to be $\frac{1}{5}$.

§ 40. Multiplication of Common Fractions.

Much of the difficulty experienced by learners in dealing with fractions has arisen from the paradoxical use made in regard to them of the words *multiplication* and *division*. The idea suggested in common language by the first of these words is that of increase or making many-fold, and by the second, that of diminution; but among fractions the rules given for operating produce results directly the opposites of these. Thus, while 6 integers multiplied by 4 give the product 24, as would be expected, the same integer number, said to be multiplied by 1 reproduces only itself, or 6; and 6 said to be multiplied by the fraction $\frac{1}{3}$ produces only 2.

§ 41. Multiplication, when defined to be—the taking a number called the multiplicand as many times as there are units in the

number called the multiplier, is done truly by taking 8 four times and making the product 32, but not truly by what is called multiplying 8 by the fraction $\frac{1}{4}$ and getting a product of 2. If the definition used for multiplication were, to take the multiplicand as many times, or *parts of a time*, as there are units in the multiplier, the definition would accord with the fact. Then 8 taken ($\frac{1}{4}$) one fourth of a time would give 2.

§ 42. The rule commonly given for so-called multiplication among fractions, is to multiply together the numerators for the new nominator, and similarly the denominators for the new denumerator. Thus $\frac{2}{4}$ multiplied by $\frac{2}{8}$ gives $\frac{4}{32}$ which, reduced to the lowest equivalent terms is $\frac{1}{8}$ (one eighth), a quantity much less than either of the factors. There is here evidently not multiplication, in the common sense of the word, but a great diminution—more parts, with less value. The operation produces really the fraction of a fraction, or the fourth part of a half. The number 20 thus acted on by the fraction $\frac{3}{4}$ gives the number 15; and so for other cases. *Subdivision,* or even *fractionising,* would appear a more befitting term for this than multiplication. The remark has been made that mathematicians, no less than lawyers, deal in fictions.

§ 43. The so-called multiplication of a number, whether integer or fractional, by a fraction, is an operation compounded of multiplication and division. In multiplying 6 by $\frac{3}{4}$, for instance, the first change is to multiply 6 by 3 wholes, making 18 wholes, but the true multiplier being only the fourth part of 3, the first product is four times too great, and has therefore to be reduced by dividing it by 4. The same view has to be taken of the multiplication of a fraction by a fraction. Suppose $\frac{2}{4}$ to be multiplied by $\frac{3}{4}$, the first act of multiplying $\frac{2}{4}$ (two fourths) by 3 makes $\frac{6}{4}$ (six fourths), which is four times too great, for the true multiplier is not 3, but the fourth part of 3. The first product, therefore, has to be divided by 4, and this is effected by multiplying its denominator by 4 (see § 35).

§ 44. Division of Common Fractions.

As a fraction is multiplied by a whole number when its numerator is multiplied by that number, so is a fraction divided by a whole number either by dividing the numerator by it, or by multiplying the denominator by it (§ 35).

When a fraction is to be divided by a fraction, meaning to show how many times or parts of a time the one is contained in the other, the rule given in books is to invert the divisor, and then to proceed as in multiplication. Thus, the multiplication of $\frac{2}{4} \times \frac{3}{5}$ is $\frac{6}{20}$; the division according to the same view would be $(\frac{2}{7} \times \frac{5}{3}) = \frac{10}{12}$ or $\frac{5}{6}$.

§ 45. It is obvious that if two fractions are reduced to a common denominator, the dividing one of the numerators by the other declares how many times or parts of a time the one is contained in the other. Thus, the fraction $\frac{6}{10}$ divided by the fraction $\frac{2}{10}$ gives as the quotient 3.

§ 46. Decimal Fractions.

In operating with common fractions, much waste of labour and time arises from the necessity, explained in § 51 and § 52, of reducing the fractions employed to others having the same denominator, without alteration of their values. This difficulty is avoided by giving, as denominator, to all the fractions occurring, the number ten (10) or some of its powers, 10, 100, 1000, &c.

§ 47. To change a common or vulgar fraction into a decimal is easy. The computer has first to multiply both the terms by ten or one of its powers, which proceeding does not alter the value; and this is done by merely adding one or more ciphers to each of the terms. He thus converts the fraction $\frac{2}{4}$ into $\frac{20}{40}$, without changing the value (§ 33). He then divides both terms so altered by the denominator of the original fraction—in the present case by 4—thus, $\dfrac{4)20}{4)40}\Big(\dfrac{5}{10}$ which still leaves the value of the fraction unchanged, and shows $\frac{5}{10}$ (five tenths) to be the decimal equivalent of $\frac{2}{4}$, both of which are expressions for one half of a whole.

§ 48. In the example of $\frac{2}{4}$ here given, the two terms, when decimalized, by being multiplied by ten, are both divisible by the denominator (4) of the original fraction, without a remainder, and the operation is thereby completed.

§ 49. Often, however, this degree of simplicity does not occur, and instead of multiplying only by 10, it is necessary to multiply by 100 or 1000 or more, that the new denominator may be a number divisible, without a remainder, by the denominator of the vulgar fraction. Thus, if the vulgar fraction

be $\frac{3}{4}$, and the first decimal multiplication consequently $\frac{30}{40}$, this divided by 4 would become $\frac{4)30}{4)40}\left(\frac{7\frac{1}{2}}{10}\right.$ or 7 and a half tenths, and therefore would still involve a common fraction; but if the decimal multiplier is made 100, the second power of 10, instead of 10 the first power, the new fraction is $\frac{300}{400}$, and this divided by 4 is $\frac{4)300}{4)400}\left(\frac{75}{100}\right.$ which $\frac{75}{100}$, reduced to lower terms by dividing both 75 and 100 by 25, gives $\frac{3}{4}$, the original fraction, and, therefore, is a true decimal equivalent of it.

§ 50. In many cases, it is necessary to multiply the terms of the vulgar fraction by adding still more than two cyphers to them, to be able to divide by the denominator without leaving a remainder; and in some cases a perfectly exact division is not at all attainable; but, as after three decimal cyphers are used, the question concerns but thousandth parts of the value of the fraction, further division is seldom important.

§ 51. The great advantage of the decimal fraction is, that it becomes unnecessary to write down the denominator of the fraction, because it is known to be ten or a power of ten, having as many cyphers following the commencing figure, as there are decimals in the nominator; thus 8·5 means $8\frac{5}{10}$; 8·624 means $8 + \frac{6}{10} + \frac{2}{100} + \frac{4}{1000}$, or $8\,\frac{624}{1000}$. The figures of the decimal numerator, therefore, can be placed in the same line with the figures of preceding integers, and are distinguished from these only by a point called the decimal point, placed between the integers and them. Thus, instead of writing the compound number $48\frac{9}{10}$ with the denominator of the fraction appearing, the same value is signified by the decimal expression 48·9. Instead of $48\frac{75}{100}$ stands the decimal 48·75. Instead of $48 + \frac{346}{1000}$ is written 48·346.

§ 52. It will be seen immediately, that decimal figures so written may in almost all computations be treated as simple integers, their exact signification being settled in the end by the required placing of the decimal point. And thus the operations of addition, subtraction, multiplication, and division, producing fractional expressions, may be carried on as if no fractions were present.

§ 53. It is important to remark, that the adding or subtracting of cyphers at the end of a decimal expression, does not at all alter its value, although the appearance and name are changed. Three tenths ($\frac{3}{10}$) is an equivalent fraction of $\frac{30}{100}$ or $\frac{300}{1000}$, and

so forth; and therefore 8·3, and 8·30, and 8·300 are all equivalent expressions. The computer who adds a visible cypher to the numerator-part of the fraction, virtually adds a cypher also to the unwritten and unseen denominator below.

§ 54. By shifting, in a row of figures, the decimal point one move to the right hand, the whole expression, and every figure in it, is multiplied by ten. By similarly shifting the point one move to the left hand, the mass of figures is divided by ten. Thus, the expression 28·46 means twenty-eight wholes and forty-six hundredths. The same figures, with the point shifted one move to the right (284·6), means 284 wholes and six tenths, and the same figures with the point moved to the left (2·846), means two wholes, and eight hundred and forty-six thousandths. Shifting the decimal point two moves multiplies or divides all by 100; three moves multiplies or divides all by 1000, and so on.

§ 55. Any decimal fraction may be converted into an equivalent common fraction, by writing the figures of the decimal denominator under the decimal numerator-figures, thus—

$$20{\cdot}5 = 20\tfrac{5}{10},\ 20{\cdot}52 = 20\tfrac{52}{100},\ 20{\cdot}627 = 20\tfrac{627}{1000},\ \&c.$$

Decimal fractions may be added, subtracted, multiplied, and divided on the same principles as common fractions.

§ 56. Addition of Decimals.

Rule.—Place the sums as if they were ordinary integers, only with their decimal points in the same vertical line, and add them as integers. To all decimal sums there may be given, without altering the value, the same number of figures as the longest has, by adding cyphers at the end of any one shorter than the rest.

Examples.

Add	25·852	7·526
	32·71	9·43
	9·406	5·500
	67·968	22·456

§ 57. Subtraction of Decimals.

Rule.—Prepare and work as for subtraction among integers, paying attention to the placing of the decimal point as for addition.

Examples.

94·928	62·208
22·732	34·672
72·196	27·536

§ 58. Multiplication of Decimals.

Decimal fractions, although their denominators do not appear written down, bear the same relations to integers that their equivalent common fractions do, and the arithmetical operations among them are in principle identical. The same paradoxical use of the word multiplication (in reality a subdivision) is made with respect to them as with respect to common fractions. For multiplication of common fractions, the rule given is to multiply together the numerators for a new numerator, and the denominators for a new denominator (see § 43). This may be done in decimals by so multiplying together the visible numerators for a numerator, and the unwritten denominators, consisting of the figure 1 and as many cyphers as there are decimal places in both denominators, as a denominator. But a shorter rule is, to multiply the two decimal expressions together, as if they were whole numbers, and then to mark off in the product, as many decimal places as there are in the multiplicand and multiplier together. This is explained by the subjoined example.

$$
\begin{array}{lr}
\text{Multiply} & 14{\cdot}86 \\
\text{by} & \underline{4{\cdot}5} \\
 & \underline{7430} \\
 & \underline{5944} \\
 & 66{\cdot}870
\end{array}
$$

By disregarding in the upper factor or multiplicand 14·86 the decimal point for 100, that factor is multiplied by 100 (see § 54), and therefore the product becomes 100 times too great. Then by disregarding the decimal point for 10 in the second factor or multiplier 4·5, the product becomes still further, ten times too great, and, therefore, to be rendered true, must be divided both by 100 and by 10, that is, by 1000 in all. This is done by placing the decimal point where it makes three decimal figures in the final product.

§ 59. Division of Decimals.

It has been explained that common fractions having the same denominator, are divided by one another through merely dividing their numerators as common numbers, omitting altogether the denominators. Decimals have already like denominators, which become exactly the same by equalising the cyphers in them. If a common fraction be left after such division, it can be converted into a decimal by the rule already given in § 47, 49. Here is a simple example:—

Divisor.	Dividend.	Quotient.
6·72)	20·16	(3
	2016	
	0	

§ 60. Involution and Evolution.

These are the names of important forms of multiplication and division, both among integers and fractions. Involution is the successive multiplication of a number by itself, producing what are called its powers, of which the second is named the square, and the third the cube, as explained at page 703, and at § 23 of this supplement. Thus, in regard to the number 2 multiplied by itself, there come—

2, 4, 8, 16, 32, 64, &c.

and in regard to 10 there come—

10, 100, 1000, 10,000, &c.

Evolution is the reverse operation, to find any root of any given number.

For small numbers the operation is simple and easy. In books of arithmetic, tables are given of the first ten powers of the first ten numbers, saving vast labour to ordinary computers.

It would be a departure from the promised simplicity of this popular Introduction to Physics to enter into the details of these operations, which would include the subjects of progressions, logarithms, &c. But essential to the purposes of the work is the consideration of the subject of proportion and the rule of three, which leads to the formation and use of arithmetical formulæ, and which can be satisfactorily explained to common apprehension.

§ 61. Proportion.

In the whole field of mathematical labour, there is no operation more valuable than that which concerns proportions. It relates, for instance, to the finding a fourth number or quantity, having a certain relation to any one of three others which may present themselves, or are already known. It has been called, therefore, the *rule of three*, and, from its singular utility, has received also the name of the *golden rule*. It is, in reality, an easy application of the simple doctrines of fractions, already explained.

§ 62. Different numbers in pairs have different relations to one another. Among the most simple of these, and the most familiar, from having to be so often referred to, are, the cases where one is the half of the other, as in the instances of

1 and 2
2 „ 4
3 „ 6, &c.

or where one is the third part of the other, as—

1 and 3
2 „ 6
3 „ 9
4 „ 12, &c.

or, placing the larger number first, when one is double, triple, quadruple, &c., of the other.

§ 63. The most clear and precise written expression for related numbers is to place them in the form of fractions, as, ($\frac{2}{3}$) two thirds, ($\frac{4}{6}$) four sixths, ($\frac{9}{10}$) nine tenths, &c., in which form the lower term declares into how many equal parts some number is supposed to be divided, and the upper term tells how many of these parts are taken to form the fraction. The pairs of numbers presented in the last paragraph may be written down as fractions, thus, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{2}{6}$, $\frac{4}{12}$; and the upper terms of all these are halves or thirds of the lower terms. All those in each set, therefore, are of equal fractional value, that is to say, are equal fractions. The relation of numbers is called their *ratio*, or proportion to each other. In the first set presented above, the ratio is called that of one to two, or of half to whole. In the second set it is that of one to three, or of a third to the whole.

§ 64. Two pairs of numbers such that between the terms of the

pairs there is the same ratio, are said to be proportioned to each other, or simply to be *proportionals*. Any two pairs of either set of those above shown are proportionals, and may be written thus, $\frac{2}{4} = \frac{5}{10}$ or $2 : 4 :: 5 : 10$, as explained in page 703, and the common enunciation in words is, "As the number 2 is to 4, so is the number 5 to 10."

§ 65. Every schoolboy is familiar with this form, as being that called the rule of three, and he knows that when the three first are placed in order, as—

$$3 : 4 :: 12 :$$

if he multiply together the second and third numbers, and divide the product of these by the first, he obtains as quotient a fourth number, having the same relation to the third that the second has to the first. This is the process,

```
3 : 4 :: 12 : [16]
          4
       3)48(1
         3
         --
       3)18(6
         18
         --
          0
```

The new number here found is enclosed in brackets. One sees that the fraction $\frac{3}{4}$ is equal to $\frac{12}{16}$ ($\frac{3}{4} = \frac{12}{16}$).

§ 66. Simple as this arithmetical operation is, the reason is not at first clearly perceived why the answer or result obtained by the rule must always be rigidly correct. The explanation is as follows.

§ 67. On considering the case here given, of $3 : 4 :: 12 : 16$, it is evident that the second term 4 is greater than the first, 3, by exactly a third part of the first; for that part, when added to the first, makes it equal to the second. If, therefore, a third part of the third term 12 (viz. 4) be added to it, making the sum 16, that sum will have the same relation to 12 that 4 has to 3.

68. Another view of the case is this. As 3 is to 4, so must 12 times 3 be to 12 times 4, for this is merely multiplying the two terms of a fraction by the same number (see § 33). Now 12 times 3 being 36, and 12 times 4 being 48, the fraction $\frac{36}{48}$ must be equal to $\frac{3}{4}$, as thus stated ($\frac{3}{4} \times \frac{12}{12} = \frac{36}{48}$). And then the two terms of the new equivalent fraction being divided by

any one number, will produce still another fraction of the same value. Therefore, dividing 36 and 48 by 3 $\left(\begin{smallmatrix}3)\overline{36}(12\\3)\overline{48}(16\end{smallmatrix}\right)$ will leave 12 and 16, having to each other the same ratio as 3 to 4. The fraction $\frac{12}{16}$ is therefore proportional to $\frac{3}{4}$, or $3 : 4 :: 12 : 16$.

This view may be stated in fewer words. If both terms of the fraction $\frac{3}{4}$ be multiplied by 12, an equivalent fraction $\frac{36}{48}$ is produced, and if both terms of this new fraction be divided by 3, another new equivalent fraction, $\frac{12}{16}$, is produced, by which the question is solved. The rule of three, therefore, which directs to multiply together the second and third of any three numbers following one another, and to divide the product by the first, offers as the quotient a new number, having the same ratio or proportion to the third which the second bears to the first.

§ 69. It is often convenient to substitute letters for the numbers in stating the terms of a proportion, thus—

$$\begin{array}{c} 3 : 4 :: 12 : 16 \\ a : b :: c : d \end{array}$$

and the mode of working the rule of three may then be thus shortly exhibited, $\frac{c\,b}{a} = d$

§ 70. The important fact has to be noticed here, that of four such proportionals as above described, the two middle or mean terms multiplied together, give the same product as the two extremes so multiplied. Thus, in the example above given,

$$\begin{array}{c} 3 : 4 :: 12 : 16 \\ a : b :: c : d \end{array}$$

3 multiplied by 16 give 48, and 4 by 12 give 48, or $a\,d = b\,c$.

Obviously, if b be the tenth (or other) part of a, and d, the tenth (or same other part) of c, the whole of c multiplied by the tenth of b must be equal to the whole of a multiplied by the tenth of c.

§ 71. Formulæ.

Having now explained generally the nature of the great fundamental operations of arithmetic, there remain to be considered the formulæ, often in the form of equations, which guide us in applying these operations in the computations made for practical purposes.

§ 72. Previously we shall note that besides the sign of the oblique cross ($\times$) used for multiplication, as explained in page 707, the same purpose is served where letters are used to represent numbers, by simply placing the letters together as they stand in printed words. Thus if a be used for 5 and b for 4, ab signifies 4 times 5, or 20, and not, as might be supposed, the mere addition of 5 to 4, making the sum of 9.—And with respect to the sign of equality (=), it is to be noted that it signifies, the equality not of the two letters or numbers placed closely on opposite sides of it, but of the whole compound expressions or series of numbers so placed. Such formulæ, called equations, are of singular power and importance in facilitating complex computations. They may be compared to a weigh-beam with the two scales balanced while charged with a variety of known weights, among which some unknown are at first mixed. The amount of the unknown is gradually discovered by shifting from one side to the other, or wholly withdrawing, equal known parts without disturbing the equipoise, until at last only what is known is left in one scale and what was unknown in the other, now ascertained by the known counterpoise.

§ 73. *General Proportion.*—When there are two particulars having such mutual relation in amount, that if one of them be increased or diminished in any degree, the other is changed in the same degree—as the weight of a moving body and its quantity of matter, the two are said to be directly proportioned to each other; and if letters are used to indicate the particulars, as W for weight and Q for quantity of matter, these in writing are connected by placing between them the sign of proportion, two dots (:) thus, W : Q, or the sign of equality (=) thus, W = Q. The use of such proportional expressions is to enable certain new results to be deduced from certain others already accurately ascertained by experiment.

§ 74. Now with respect to motion and rest among bodies, universal observation has shown to be true what is stated in this work (see the *Analysis*, page 33), that "to change any present state of motion or rest in a body, force proportioned to the change is required, whether to give motion, to take it away, or to bend it." This is a general expression for what have been called the *laws of motion.*

§ 75. To express shortly a large number of the facts, such initial letters as the following are employed:—

Q Quantity of matter.
M Quantity of motion or Momentum.
S Space passed through.
T Time elapsed.
V Velocity or speed.
P Power or force.
R Resistance.

§ 76. To compare given amounts of all such particulars, it is necessary to choose standard units of comparison that can be numbered—as ounces or pounds for quantity of matter, feet or inches for length or distance, seconds or hours for time; for to talk of dividing space by time, or multiplying weight by velocity, has evidently no definite meaning.

In pages 53 and 54 are given many examples, showing relations among Quantity of Matter, Force, Velocity, Momentum, &c.

An arithmetical expression for momentum is $M = Q \times V$ or $M : Q\,V$, signifying that momentum is proportioned to the quantity of matter and the velocity conjointly.

§ 77. In respect to Velocity, V, the degree depends on the relation between the space passed through and the time elapsed during the passage. If, of two bodies, one move three feet in a second, and the other two feet in the same time, the velocity of the first to that of the second is as 3 to 2. In equal times, the velocities are as the spaces, $V : S$; but if the times be unequal, the velocities are as the units of space divided by the units of time. Thus if one body pass over 10 feet in 2 seconds, and another pass over 8 feet in 4 seconds, the velocity of the first is to that of the second as $\frac{10}{2}$ to $\frac{8}{4}$ or as 5 to 2; for, by dividing 10 feet by 2 seconds, the speed of the body is seen to be 5 feet in one second, and by dividing 8 feet by 4 seconds the speed of the other body is seen to be 2 feet in one second, and their respective velocities therefore are as 5 to 2. This fact is expressed by the formula $V = \frac{S}{T}$: the velocity is as the units of space divided by units of the time. The velocity of motion, therefore, through a given space in a given time, is greater just as the time elapsed is less, and is expressed by the quotient of the units of space divided by the units of time.

§ 78. From the formula $M : Q\,V$, meaning that the momentum

or quantity of motion in a body is proportioned to the quantity of matter multiplied by the velocity, it may be deduced that a body of 10 pounds weight, moving with a velocity of 10 feet in a second, has the same momentum as a body of 5 pounds moving at the rate of 20 feet in a second; for 10 multiplied by 10 make a hundred, and 5 multiplied by 20 make a hundred ($10 \times 10 = 100$, and $5 \times 20 = 100$). (Art. 139.)

§ 79. That the quantity of motion in a body is proportioned to the force producing it ($M = F$), is shown in the fact, that the velocity of a falling body increases in exact proportion to the time during which the force of its weight or gravitation has been allowed to act upon it.

§ 80. From pages 101 to 105, the very important fact or principle is described in detail, that a small weight attached to the arm of a lever, or weigh-beam, at a given distance from the axis or fulcrum, has force to balance a greater weight or resistance acting on the other arm at a shorter distance from the fulcrum; and that, for exact balance, the smaller weight must be as much further from the fulcrum than the other, as the other is weightier than it.* If the small weight be 2 pounds and its distance 8 feet, and the greater weight be 4 pounds, the distance of that must be 4 feet. The letter P may indicate the small weight or power, R the greater weight or resistance, p the distance of the small weight, and r the distance of the larger. Then the units of weight and distance P, p, multiplied together must always give the same product as the units of R r. The facts thus expressed have the form of what is called an equation, $P p = R r$; and any three of the particulars being given, the remaining one may be found by working the rule of three in regard to it, thus:

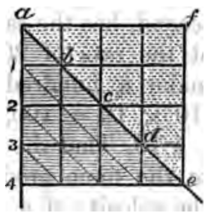
$$P = \frac{R r}{p} \qquad r = \frac{P p}{R}$$

$$R = \frac{P p}{r} \qquad p = \frac{R r}{P}$$

§ 81. Such relations among quantities as here spoken of, are interestingly exemplified in the case of a body set free to obey the force of gravitation during its fall to the ground. This phenomenon is fully analysed in the pages from 61 to 65, and is reviewed in page 716. As these supplementary pages are printed separately, the illustrative diagram is here repeated.

* At page 103, line 18, the letter c is to be placed instead of b.

The perpendicular line *a* 4 marks the progress of a fall during four seconds of time, divided into smaller instants. The horizontal lines across from this perpendicular to the oblique line *a e* represent the velocities, uniformly increasing at every instant, and which, at the end of the first second, has been ascertained to be at the rate of 32 feet (and a fraction) per second, the average velocity for the whole second being 16 feet, reached at the half-second. The darker triangular space or area, bounded by the lines *a* 4 and *a e* at the sides, and by one of the horizontal lines below, is a measure of the amount of fall or space passed through at the instants marked by the horizontal line of velocity. The area of the first small triangle *a* 1 *b*, belonging to the first second of time, indicates the 16 feet of fall at the end of the first second. Any point in the line of time, *a* 4, marks a certain instant of the fall; and any line crossing from such point to the diagonal *a e*, tells the velocity at that instant; while the shaded space above the line measures the amount of fall at the instant.



At page 62, the reader learns—

1. That the velocity of a falling body increases uniformly at every instant.
2. That the body falls 16 feet and a fraction in the first second.
3. That the extent of fall increases as the square of the units of time.

If the question be put, therefore, how far a body will fall in any given time, as three seconds, the following formula, when changed into numbers, gives the answer:—

S represents the space fallen through in feet.
T the time in seconds.
g the force of gravity, causing a velocity of $32\frac{1}{6}$ feet per second, and a fall of half that ($\frac{1}{2}g$) during the first second.

Formula.

$$S = \tfrac{1}{2}g \times T^2$$

This becomes $S = 16 \times 3^2$ (or 9)

and this $S = 16 \times 9 = 144$

declaring that the body in falling freely descends 144 feet and a fraction in three seconds.

General Remarks.

On Natural Science, or knowledge of the forces which produce all the changes and results going on around men on earth; and on the extent to which men have come to understand these forces, so as to be able to modify their agency for useful purposes.

* 1. Children who during successive years have sported under the shade of the same great tree, are seldom aware that the tree, like themselves, has been growing all the while; so the mass of people in the world are not aware that, in the midst of very evident political fluctuations, there is going on a decided progress, on the whole, towards a higher state of well-being, of which the limit is as yet but imperfectly conceived.

* 2. Within the last weeks of the year 1866, two events occurred of an importance which has strangely awakened public attention to this subject of progress. The first was, after failure of former attempts, which seemed to forbid hope of success, the final laying down of two distinct electric cables across the bed of the Atlantic Ocean, in some places more than three miles deep (forty times the height of St. Paul's Cathedral), so satisfactorily, as to remove dread of future failure. Through these precious wires, persons, thousands of miles apart, can now converse almost as readily as if they sat at opposite sides of a table. The second marvel was, that chiefly through the introduction of a simple mechanical improvement in fire-arms, the much less numerous population of Prussia overcame the vast military power of the Austrian empire. It would appear from this, that henceforth countries will be powerful, safe, and prosperous, in proportion as they attain knowledge of the laws of inanimate nature, that is, of physical science, with practical skill or arts

grounded on that, by which they may direct the energies of nature to produce results useful to them.

* 3. This view is powerfully confirmed by reflecting that when many of the persons still living were born, there was not in existence a Watt's steam-engine, such as now, in thousands, are doing man's work all over the earth; nor was there a gas-light, nor a railway, nor a steam-ship, nor electric telegraphs, nor photographs, nor others of the things which, within a century, have changed entirely the condition of the human race on this globe.

* 4. Persons may at first feel surprise that the great inventions referred to have been so long delayed; but the explanation is complete when a few known facts in the political history of Europe are considered.

* 5. It is known that from about four thousand years ago, the Greeks first, and then the Romans, were making advances in many of the arts of civilization, far exceeding those of other nations. Their architecture, sculpture, and painting attained excellence which has not been surpassed, if equalled, in modern times; and in the horrid work of war, offensive and defensive, they seem to have accomplished all that was possible before the invention of gunpowder. In the copiousness and polish of their languages, they advanced from the rude forms of earlier times to what, as exhibited in their poetry, eloquence, and historical narratives, has appeared to modern judges to be perfection.

* 6. Strangely, however, their opinions in regard to the origin of the universe, and to theology, appear to present intelligence to be almost incredibly absurd or irrational. They believed that this world had not existed very long, and that present events in it were dependent on the will, caprices, and often low animal passions of beings whom they called deities, or gods and goddesses, who were immortal, but in other respects so nearly resembled human beings that the races might occasionally mix and multiply. They spoke of Jupiter as the father of their gods, commanding obedience by launching his thunderbolts; Neptune they believed was ruling the waves, Eolus the winds; Mars was supposed to decide the event of battles, and so forth. In many situations, magnificent temples had been raised to serve in the worship of these deities under the superintendence of a venerated priesthood. Little at that time did the people dream that in future ages their descendants, although mere men, would acquire power over nature superior to that attributed then

to their gods; such, for instance, as the using lightning, under the name of electricity, to carry messages to distances of thousands of miles, even across broad seas, and to bring back instantaneous answers; or the constructing of ships driven by steam, cleaving the waves more swiftly than ships with sails had ever done, and able to set at defiance the stormy winds and waves; and, lastly, constructing artillery vastly more destructive than any means ever attributed to Mars.

* 7. The Greeks and Romans believing, as they did, that events on earth depended on the will of such deities as here described, of whose will they could have no foreknowledge, were prevented from thinking that by studying the ordinary course of nature they might detect, as their descendants have done, certain unchanging laws of sequence among the phenomena, the knowledge of which would enable them often to foresee what was coming, and to direct the agency of the laws to effect new results of the highest importance to their own well-being.

* 8. Two thousand years ago the Romans had obtained dominion over nearly the whole world then known to them. It was then that Julius Cæsar, returning to Rome from successful wars in Gaul and elsewhere, subverted the government of the senate, and became absolute emperor, dictator, or despot. Afterwards, amidst strange fluctuations, corruption and confusion spread, and the decline and fall of the Roman power had commenced. At last, barbarous nations from the north and east, in overwhelming multitudes, broke in upon the empire, and, working prodigious slaughter and devastation, dismembered it. Different tribes kept possession of different portions, and, intermarrying with what remained of the former inhabitants, formed eventually the distinct kingdoms of modern Europe.

* 9. The pure Latin then ceased to be the common language of the different parts of the former empire, giving place to confused mixtures of the languages of the victors and the vanquished. These were little intelligible except to the occupants of the several localities. Had it not been for the fact that Christianity was then spreading over Europe, humanizing the people, the sacred books of which religion were chiefly Greek and Latin, and that the church services everywhere were Latin, the two ancient languages of Europe, with their valuable literature, might have been lost entirely, as great part of it really was.

* 10. The want of a common language in Europe, clearly

understood by the people of the different nations, was powerful in delaying the advance of a new, and what was to become, a much higher civilization, arising from new discoveries in science and new inventions in art founded on these. The noble intellect of man, although at this time obstructed, did not become torpid; and striking instances of its activity were every now and then starting up during what were called the 'dark ages' between the old and the new civilization. Thus Columbus, a Genoese mariner, saw evidence which convinced him that this earth is in form a globe, and in his first voyage he fell upon, what has since been called, the new world of America lying in his way. Copernicus afterwards proved that this earth and similar globes, called planets, are all moving in orbits round the sun. Galileo, through his newly-invented telescope, then first saw the four moons of the great planet Jupiter, wheeling round it as our moon wheels round this earth. Newton discovered the great fact of *Universal Gravitation* which, with the simple laws of motion, regulates all the movements of the heavenly bodies. Besides these sublime facts, many subordinate novelties came, as the invention of gunpowder, the art of printing, &c., leading towards the marvels of the two last centuries already referred to.

* 11. In the middle ages, the new forms of language called Italian, Spanish, French, English, German, &c., were all makng important progress, and producing admirable literatures. No one of them, however, was so generally known as to be a convenient medium for scientific intercourse between the nations. The Latin, therefore, although not deeply studied in any of the countries, was studied to a moderate degree in all, and therefore soon became the medium. The books of general interest, as those on the main branches of knowledge—of law, for instance, medicine and philosophy, were written in Latin. This practice prevailed even so late as Newton's time, who wrote his immortal *Principia Philosophiæ Naturalis* in Latin; so did Bacon his *Novum Organum.* And the lessons or lectures in the different universities of Europe were delivered in Latin. It became necessary, therefore, to establish schools everywhere for teaching Latin.

* 12. The superiority of a polished language, like Latin, over the yet imperfect forms of the vulgar tongues in Europe, was, some centuries ago, so great that in many persons, admiration of it became a kind of idolatry; the mere words and phrases, particularly of the poets, being deemed more worthy of study than the

vastly more important knowledge of actual things and scientific relations which might have been conveyed to them in their mother-tongues. On this knowledge, joined with Christianity and more correct views than formerly of social economy,—the last-mentioned still rapidly advancing, the modern civilization of the world is based. Moreover, there is, under the title of *comparative philology*, or the philosophy of language generally, a higher and more profitable study of language than in labouring on the details of any one language except those of the mother-tongue, which every person should well know.

* 13. The writer of this, because advocating so strongly the study of natural science, has been thought by some not to attach due importance to the ancient classics, but wrongly. He has much regretted the waste of time caused by faulty modes of teaching, holding strongly the opinion of Milton, himself for some time a teacher, as given in the following words of his well-known letter to Mr. Hartlib:—"We do amiss to spend seven or eight years in merely scraping together as much miserable Latin and Greek as might be learnt otherwise easily and delightfully in one year." Knowledge of Latin, besides giving access to the admirable literature of the language, confers, in the present state of things, also incidental advantages. It is essential to a student of medicine, who will have to write prescriptions in Latin. It serves as a means of obtaining scholarships at the University. It greatly aids the study of the modern languages of Europe.

* 14. When the writer of these pages went to the University, the progress in the world of new discovery in science and arts had become very rapid. Watt had just completed his improved steam-engine, and ingenious men everywhere were labouring to adapt it to the many purposes it was capable of serving. Hence in the class-room the interest had become intense of the lessons given which showed that the greater part of the phenomena of nature were consequences of a very few simple laws of motion and change, intelligible to ordinary comprehension, and which ordinary ability might afterwards apply to use.

* 15. The writer, after leaving college, was so frequently a witness of advantages arising to persons who had physical knowledge, and of evils suffered by persons who wanted it, while he believed that instruction in such matters might be rendered more easy and attractive than usual by some changes in the mode of teaching, among others, by avoiding obscure technical

language—that on his returning from abroad to settle in London as a physician, he undertook to deliver, in the Philomathic Institution near his residence, a course of lectures on Natural Philosophy and Chemistry. In the printed announcement, he pointed out the need of such knowledge to persons, not of one, but of all classes (as repeated in the Introduction to this work at page xxv.). In regard, for instance, to the medical profession, he said, Where, to illustrate mechanics, is to be found a system of levers and hinges and moving parts like the limbs of an animal body? where such a hydraulic apparatus as in the heart and blood vessels; such pneumatic apparatus as in the breathing chest; such acoustic instruments as in the ear and larynx; such optical instruments as in the eye; and, in a word, such variety and perfection as in the whole of the visible anatomy? In regard, also, to the education of women, it might be observed that in England now, not only in palaces but in ordinary dwellings, there are branching pipes for water supply, other pipes for gas, with meters and burners; then a variety of fireplaces to maintain healthful temperature in winter, arrangements for required ventilation, well-closed house drains, &c. Among these, owing to ignorance or neglect of servants, accidents frequently occur, causing great annoyance and often danger; against which there is great security where the eye of an intelligent mistress glances over the proceedings.

* 16. Professional occupation prevented the repetition of these lectures. Afterwards, however, in the various medical societies, some views were put forth by Sir David Barry and others on the circulation of the blood, which erred in stating that the force moving the current in the veins was atmospheric pressure. The author's opinion was requested, and he afterwards gave a few lectures, including that subject. Not having leisure to repeat these, he published the *Elements of Physics* for general use, written in plain non-technical language, even in dealing with mathematical relations.

* 17. That such a book was wanted, not only for England, but for the other countries of Europe, seemed proved by the new editions quickly called for, the reprints in North America, and the translations made into the principal European languages. The notices of the book by contributors to the public press gave so instructive and useful a view of the state of the public mind in regard to common education, that a few extracts from the large number are here reprinted in page 46.

* 18. When, in 1836, Government founded the *University of London,* then so much wanted, and now so signally prospering, the author had the honour to be appointed a member of the Senate. Among the labours of the senate was the arrangement of a new curriculum, or course of study, more extended than before, to be pursued by candidates for the university degrees and honours, as suited to the progressing condition of society, which now opens the way to high appointments, civil and military, at home and abroad, through competitive examinations in general useful knowledge. The natural sciences of Physics and Chemistry were there included as fundamental parts of the course. It was at first objected by some parties that more was required than ordinary students could bring to the test. But experience has since proved that this fear was vain; and in the older universities, and in some of the higher public schools, professors or teachers of experimental physics have since been appointed. It is known that science, when perfected or even only advanced, is often more easily learned than in its earlier stages. A person may now learn more of astronomy in a week than, before Newton had written, he could have learned in a year or more.

* 19. The author of these pages cordially joined his enlightened colleagues in labouring to widen the curriculum. He was requested by the senate to act on the committee which drew up the novel and important scheme of study for the degrees of *Bachelor* and *Doctor* in *Science,* to be conferred after examination in the fundamental sciences of Mathematics, Physics, Chemistry, Life and Mental Philosophy, passing more lightly over the Languages, with the exception of our own.

* 20. As marking that the author had reason to feel strongly the obligation which scholars contract towards the school and teachers where they are trained; but chiefly because he desires to awaken in the public mind a just appreciation of the importance in general education of the department of Natural Philosophy, called by Lord Bacon the root or foundation of the other sciences—he mentions the following incidents. When the class-rooms where he had studied were to be rebuilt in 1830, he returned to the building fund, the amount of the scholarship which he had enjoyed when first a student there; and when, six years ago, the two university colleges of Aberdeen were united to constitute one, he offered through Provost Webster an adequate sum to allow of the lectures on Natural Philosophy being given

where he himself had studied, near the centre of the town, some of the lectures being left open, as had been practised before, to inhabitants of the town, for a moderate fee, without matriculation.* Mrs. Arnott, with the same views and wishes in regard to the study of elementary Natural Philosophy by those of her own sex, has made endowments to two Ladies' Colleges in London—the Queen's College in Harley Street, and the Bedford Square College. And lastly, arrangements have been made that the profit arising from the sale of this work shall serve to lower its price for general use, and, if the demand for it continue, shall procure fit editorial assistance for future publication, to keep it up to the level of the state of science at the time. The entire work is now sold for the price of the first half of the work when originally published separately.†

* The circumstances of the University induced him to substitute for that offer a Scholarship of £50 in Natural Philosophy, which has been accepted by the Senatus, and is awarded at the end of the sessions.

† The last pages tell, how important in the education of a people the knowledge of Physics or Natural Philosophy is. Of its full value, however, the popular mind was so little aware, that until lately it was not a part of the prescribed course of study in our public schools and colleges. There is another portion of knowledge, also of high value, which has been similarly misappreciated, and which is therefore referred to here; namely, the fact that human enlightenment, and consequently Civilization, is in its nature slowly progressive. As a child is born into the world utterly ignorant of all that exists there, and acquires knowledge of things and of the course of change among them only according to the opportunities of observation offered and the artificial aids in teaching provided by seniors; so has the human race as a whole had gradually to advance during ages, from profound original ignorance to the now existing state of knowledge and arts. Study of the steps of past progress tends strongly to quicken further progress. The author has reviewed this subject in detail, in a volume (second edition) published with the title of *A Survey of Human Progress.* It is really a treatise on Education, which attempts through simple arrangement and language to render a subject which has been deemed obscure, so intelligible to common minds, as to serve both as a direction for the studies of individuals and as a help to those planning new legislative enactments at present required in relation to education generally.

It is interesting to contemplate the very close resemblance in bodily constitution between man and the higher orders of inferior animals, as the lion and tiger, which, like him, eat flesh for their sustenance, and then to consider the immeasurable superiority of the intellectual faculties of the human mind or soul to the feeble shadows of these, named instincts, in brute animals. One particular in the comparison is, that man has very clear foresight of much that is to happen in the future, and with his power of devising great variety of means to effect ends, he can modify greatly the course of nature to answer

innumerable purposes. Man can foretell to within a few beats of a pendulum the coming on of an eclipse, a thousand years before it happens; whereas a lion can form but a dim conception of any one day in the coming week.

Man and these inferior animals have bodies consisting of like bone and flesh, in form of external parts for locomotion and other actions, as in obtaining food, and of internal parts which convert the food into blood, to circulate through the heart and branching channels to every point. They all have a brain and nerves, and connected with these, sensory organs of touch, seeing, hearing, taste, and smell. They all sleep and wake, they all are born, and grow old, and die. They all require an unfailing supply of nourishment, fresh air, and warmth, dependent more or less on their own exertion; and if any one of these is not supplied, they quickly die.

The inferior animal, when aroused by its appetites and instincts, being short-sighted, knows no other or better mode of gratifying its want, of food for instance, than, when a fit object is seen, to seize it at once, and then to kill and devour, utterly regardless of the suffering inflicted. There is, therefore, violence, cruelty, and death preceding the repast; and there is necessarily unceasing war between the stronger creature which eats and the weaker races which are to be eaten.

Man, on the contrary, by his penetrating foresight and power of devising, joined with quick susceptibility to compassion and benevolence, sees the possibility of obtaining all he needs in unlimited abundance and at all times, through gentleness, judicious care, and general kindness, instead of through brutish violence. This is seen in that most precious of his inventions, the art of taming certain animals and raising flocks and herds. Such animals he protects from their natural enemies; he shelters them from the inclemencies of the weather, he provides for them abundant food through all the seasons. From the beginning of their lives to the end they need never have a pain, either actual or apprehended, thus exhibiting a striking contrast to the fate of animals in the wild state, called that of nature, with their fierce enemies always near them; and when at last the owner wants the body of a sheep after its life has ceased, the death may be so sudden that there has not been time to feel pain, and the creature has merely left room for another like itself to inherit its happy lot.

Then in the dealings of human beings with one another the same noble reason with foresight soon discovers to them that by practising universally perfect justice and benevolence, in lieu of the narrow selfishness of inferior animals, all painful rivalry and disagreements may be avoided, and the satisfactions of human existence may be increased illimitably. The golden rule of human conduct, easily understood by common apprehension, is—to act always towards others as one would wish to be treated if the relations of the parties were reversed.

Man with abundance of the prime necessaries of life always at command, and freed from all anxiety respecting the future, is at liberty to employ his whole time, and his high mental powers, in devising numberless new arts to multiply and enhance his enjoyments. He can gradually convert what was once a desert into a noble city like London. His powerful memory retains knowledge of the results of his own experience, and through language, he may obtain from others and from books knowledge of the most interesting occurrences which have happened in preceding time. The steady course of

nature thus revealed to him warns him of much that is to occur in time to come. Man, therefore, unlike the brute creature, which can know only what happens during its own short life in its own neighbourhood, may be said to have lived through long past ages of progressing civilization, and, in a degree, to live also through the future with higher anticipations realized.

It is to be remarked that man, although always possessing his high intellectual powers, has, when born, no more knowledge of the world into which he has come than any of the lower animals, and if left without education or training from seniors, would for a time act in obedience to mere animal wants and instincts. This has been proved where, by accident, young children have been cast and left unprotected and untaught on a fertile tropical island. They have grown to maturity, and multiplied as so many sheep or other inferior animals would. Sad it is to know that there have been many, and are still some regions of the earth occupied by men nearly in the lowest animal condition, the families or tribes being almost constantly at war with one another, aiming at extermination of rivals, with even cannibal cruelty.

Parents in a civilized community, become aware of the boundless advantages of acting towards one another with mutual kindness, that is to say, with perfect justice and wise benevolence, naturally teach this conduct to their children; and their lessons and example operating from early youth to restrain the tendencies to narrow animal selfishness, the training produces what has been called a second nature, seen among civilized as contrasted with savage men.

As civilized families become larger and multiply, the seniors see it useful to frame and establish rules or laws of conduct for all, and to appoint officers or magistrates to superintend the execution of the laws. The crowning arrangement of general civilization will be, when in every country there is established a thoroughly suitable scheme of education for the children of all the classes of the community. The approach to this in any country is not yet very near, although there seems now to be an awakening of the public mind in regard to it.

It is still a fact, however, that distinct large communities, or parties in the same, although tolerably well governed within themselves, will manifest towards one another decided narrow selfishness, even like that of savage tribes carrying on wars with the usual cruelties. Witness what has occurred lately between the States of North America, and more lately, between the northern and southern states of Germany. In America, within three years, hundreds of thousands of the people fell in mutual slaughter, as did forty thousand within three days in the end of last year in Bohemia. Had the masses of the people in these countries been trained from early youth to conceive vividly on one side the horrors of war, and on the other the advantages of peace; and had they been accustomed to obey the precept which the religion professed by them as well as worldly wisdom directly inculcates, of "doing to others as they would be done unto;" and had there existed fitly chosen representative assemblies enabling the masses to influence the governors, such wars could not have broken out.

England has had the high distinction of being the first to see clearly, and to exemplify in practice the effects between nations of freedom of commerce (free trade) tending to tie them together as a great helping brotherhood. Ignorant prejudices, and the narrow interests of particular classes, for a long time strongly

opposed everywhere, the adoption of free trade; but now, to the surprise of some, and to the profit and delight of all concerned, it does exist, to a certain extent, and is producing its happy results. In the British dominions the abolition of negro slavery and other kindred measures were preparing the way.

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* Note to Page 485.

This section of four pages relates to the important facts ascertained since the early editions of the Physics, on the subject of the mechanical dilating force of Heat and the relation of that to other forces. The attention given by the author to the subject appears in the edition of 1829, in the following lines of Vol. II., from page 69 forward.

Page 70. "If not before, at any rate soon after steam-engines began to be used, and had so strikingly shown to what important purposes the force of an expanding aëriform fluid might be applied, the thought would naturally occur that the force of common air dilating by heat might also be rendered useful."

Page 71. "The value for work of a foot of steam passing from a boiler into a working cylinder would be, to press up the piston of the steam-engine through a foot (an explanatory diagram is given) with force all the way of 15 lbs. per inch of the piston surface (page 96). Persons may not reflect that 15 lbs. on the square inch is about a ton (it is exactly 2,160 lbs.) on the square foot."

In pages 69, 70, it is stated that the bulk of any quantity of air is doubled by an increase of temperature of about 500°, which implies that a cubic foot of air dilating to double bulk under a piston, will exert exactly the same force as a foot of steam entering there. It follows from this that if the so-called capacities for heat of the two substances were accurately known, the force-value of the heat employed in the two would be accurately ascertained. But in 1829, the capacities for heat of water and air, under different degrees of compression, were not perfectly known. Some instances are given, at page 52, of the capacity diminishing with increase of pressure. At page 72 a rough estimate is made in round numbers, not meant as a direction to engineers but as encouragement to experiment. Since then the admirable researches of Joule, Mayer, and Regnault have given precision to all such calculations, and have led to the determination of what is called the mechanical equivalent of Heat, or Joule's equivalent, as recorded here, at page 486. In 1852 Dr. Joule read a paper before the Royal Society (published in the *Transactions* of the year) reviewing the whole subject of the expansion of air by heat, and describing a simple air-engine such as sketched in the *Physics* of 1829, page 72, with the burning fuel enclosed within the receiver. The order of thought and illustration used in the early editions is repeated in the new edition of 1865, with a notice of the discoveries made nearly twenty years ago by Dr. Joule and others.

NOTICE.

A strange incident may with propriety be mentioned here. In a literary journal, the 'Reader,' (now for various reasons discontinued,) there appeared in March 1864, *a detailed and favourable criticism of the 'Elements of Physics;' but in October* 1865, *under a change of editor and some new interest, an article was admitted intended to disparage the work. Not finding scientific errors in the book itself, to serve the purpose, it charged the author with having taken for his last edition, without acknowledgment, some information respecting the Mechanical Equivalent of Heat from Chap. III. of Professor Tyndall's recent work on 'Heat as a mode of Motion.' That this, which was the only specified charge of unfairness, was utterly groundless, is seen by comparing the part impugned, page* 485-6 *of the present edition, with the quotations from earlier editions, reprinted here on the last page* (43), *and with the statement of Joule's discovery of the Mechanical Equivalent, as given long ago by himself.*

It will interest students to note, in reading from pages 2 *to* 5 *of Vol. II. of the Physics, published in* 1829, *of which the substance is repeated almost verbatim in the pages* 402 *to* 405 *of the present edition, that the author at that time considered heat to be an undulatory motion among particles, connected with similar motion in an elastic imponderable medium pervading general space; and further held, that there was intimate co-relation in nature between the phenomena and forces of Heat, Light, Electricity, and Magnetism.*

EXTRACTS FROM RECENT REVIEWS OF THE *ELEMENTS OF PHYSICS.*

THE ATHENÆUM, 14th Oct., 1865.

. . . We know of no work which so faithfully fulfils the duty of supplying general readers with elementary matter as the Physics of Dr. Arnott. It is written in a remarkably clear style; and often when dealing with the more exalted truths it rises into eloquence without being inflated.

THE FORTNIGHTLY REVIEW, 1st Oct., 1865.

. . . That after so long a delay there should still be a public eager for Dr. Arnott's new edition is a striking testimony to its reputation; and indeed we may say, without injustice, that the qualities which many years ago gave to this treatise a distinctive place among works of popular science, still give it a superiority over all rivals. In perfect clearness of conception, and consequent clearness of statement, Dr. Arnott has not been surpassed. His book may be read by a philosopher or a child. . . .

THE LANCET, 14th Oct., 1865.

. . . The charming simplicity of the style, and the erudition which it displayed, gave it at once a world-wide reputation. It was translated into most European languages, and reprinted in America. The revolution which has occurred with respect to many subjects discussed has afforded the learned author the opportunity of embodying in these volumes what is novel and interesting. . . .

DAILY NEWS, 14th Oct., 1865.

. . . Dr. Arnott's Physics is honourably distinguished as the first work of serious value in which the attempt was made to convey in the English tongue the leading facts and doctrines of science without employing the technical phraseology of the schools. In our own days everybody is expected to have a general acquaintance with them; and this change is in a great measure owing to his writings. . . .

THE MEDICAL TIMES AND GAZETTE, Jan., 1866

. . . There is a charm in this work depending on the simplicity with which the principles are laid before the reader, and an enthusiasm which pervades the author's exposition of the wonders of nature which render it extremely popular. . . .

THE LONDON REVIEW, 17th Feb., 1866.

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